Homework 4
due Wednesday February 1, 2017

Do these problems, and email the results (if you can, in a PDF or else in Word .DOC or .DOCX format. PDF is best as I cannot read some .docx files). Like all of the weekly homeworks this one totals 25 points and is due at the end of that day (at midnight).

1. (13 points) A clonally-reproducing organism with two types of clones (A and a) needs nest holes to reproduce, with each parent using one hole. There are two sizes of holes available, small and large. There are equal numbers of small and large holes. Type A can use large holes but not small holes, and produces 30 offspring per parent. Type a prefers small holes, and produces 20 offspring per parent in them. But when they are not available it competes with A parents for large holes, but in them it only produces 10 offspring per parent. Figure out what happens to the gene frequency (clone frequency) for this case of frequency-dependent selection by following these steps. Assume that there are a lot more potential parents than nest holes.

   (i) If the gene frequency of A in the parents is p, assume that all small nest holes are filled by a individuals. Then, if we assume that all the remaining parents are equally likely to end up in large holes, imagine that a fraction p of those large nest holes being filled with A clones, and $1-p$ of them filled with a clones. *This is an approximation. When there are many more parents than nest holes, one can assume that when the small nest holes are filled, there are still almost $1-p$ of the remaining individuals that are type a individuals among those left to compete for the large nest holes. One can do this more exactly, but that makes this problem harder so I don't ask for that calculation.*

   (ii) From the numbers of offspring given, calculate what fraction of all of the population's offspring will come from A clones, and what fraction are from a clones? Explain.

   (iii) Work out how the gene (clone) frequencies change. What is the formula for the fraction of A clones as a function of the fraction in the preceding generation? Will there be an equilibrium? Is it stable?

   (iv) Is the equilibrium frequency the one where the population has highest fitness? Show the fitness calculation.

2. (12 points) Suppose that a rare allele A in a population predisposes the carrier to have a behavior that decreases the individual's fitness by 10%. The behavior affects 10 other individuals, decreasing each of their fitnesses by 20%. The average coefficient of relationship of the individual to these 10 others is r.

   (i) Considering the fitnesses of these individuals, including the individual and the recipients of the behavior, as well as the fitnesses of the other individuals in the population, will the frequency of this allele be expected to increase? In terms of Hamilton's Rule, why or why not?

   (ii) How does the rate of increase or decrease of the gene frequency of this alleles depend on the coefficient of relationship r? Explain why,

   (iii) In the section of chapter II on kin selection, I list a number of different cases in which the fitnesses are affected in different ways. Is this one of them. Explain why or why not.